

# DOINGWHATWORKS



SAMPLE MATERIAL

## Focus on Fraction Operations

RMC Denver Professional Development

**Topic:** Developing Effective Fractions Instruction for K-8

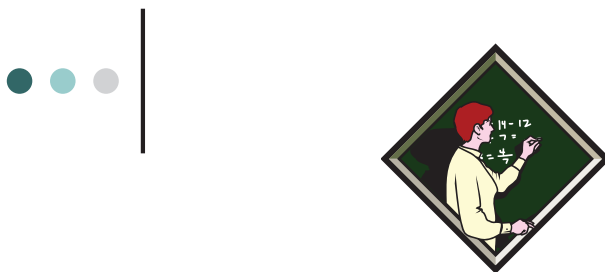
**Practice:** Operations With Fractions

As part of a week-long professional development workshop provided by RMC Denver, teachers spend a day focusing on fraction operations. The media piece *Multiply or Divide?* is one session in the Focus on Fraction Operations day of the workshop.



# Focus on Fraction Operations

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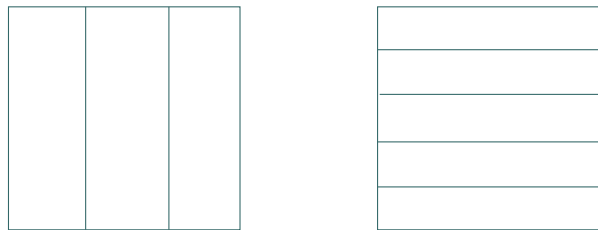


## Addition and Subtraction of Fractions

## Addition of Fractions

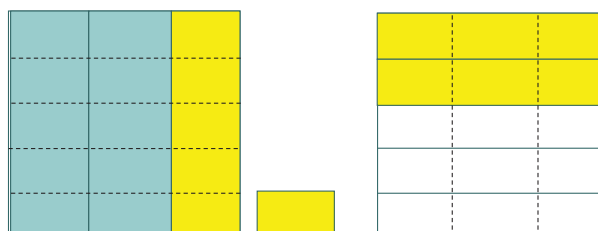
Area model for addition

$$\frac{2}{3} + \frac{2}{5}$$



## Addition of Fractions

Area model for addition:  $\frac{2}{3} + \frac{2}{5}$



Make horizontal cuts on vertical and vertical cuts on horizontal.  
Move the pieces from one rectangle to other rectangle.

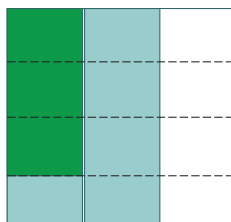
**Answer is 1  $\frac{1}{15}$ .**



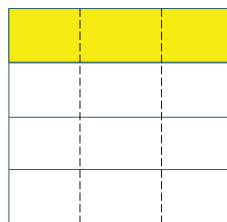
## Addition and Subtraction

Does area model work with subtraction?

$$\frac{2}{3} - \frac{1}{4}$$



**2/3 of a rectangle**



**1/4 of same rectangle**

Make horizontal cuts on vertical and vertical cuts on horizontal.  
Remove 3 of shaded pieces from 2/3 rectangle. Five pieces remain.  
**This is 5/12 of rectangle.**



## Addition of Fractions



Consider a common misconception related  
to addition of fractions:

$$a/b + c/d \text{ becomes } (a+c)/(b+d)$$

For example, a student might write

$$\frac{5}{8} + \frac{5}{12} = \frac{10}{20} = \frac{1}{2}$$

## ● ● ● | Addition with Fractions

**Possible explanations for this method:**



Student A: We add things that are alike, so add the things that are the same (alike) in fractions: add numerators and add denominators.

Student B: For multiplication of fractions, we multiply numerators and multiply denominators, so for addition we add numerators and add denominators.

Student C: It's easy to just add numerators and add denominators.

Student D: It's like in a basketball game. In the first half of the game, the player makes 5 out of 8 shots. In the second half, the player makes 5 out of 12 shots. So, in the whole game the player makes 10 out of 20 shots or  $\frac{1}{2}$  of the shots.

## ● ● ● | Addition with Fractions

**How do you respond to these explanations?**

- Reasonable?
- Role of context?
- Role of unit in adding fractions?
- What would happen if we defined  $a/b + c/d = (a+c)/(b+d)$  ?





## Multiplication and Division of Fractions



### Multiplication of Fractions



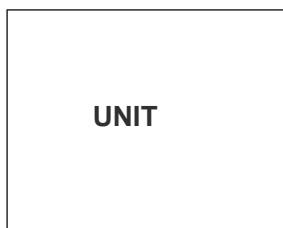
#### No Computation algorithms – Drawings only!

- You have  $\frac{3}{4}$  of a pizza left. If you give  $\frac{1}{3}$  of the leftover pizza to your brother, how much of a whole pizza will your brother get?
- Someone ate  $\frac{1}{10}$  of the pan of brownies before putting it on the table. If you eat  $\frac{2}{3}$  of the pan of brownies that is left, how much of the whole pan of brownies will you have eaten?
- Gloria used  $2\frac{1}{2}$  tubes of blue paint to paint the sky in her picture. Each tube holds  $\frac{4}{5}$  ounce of paint. How many ounces of blue paint did Gloria use?
- Zack had  $\frac{2}{3}$  of the lawn left to cut. After lunch, he cut  $\frac{3}{4}$  of the grass he had left. How much of the whole lawn did Zack cut after lunch?
- The zookeeper had a huge bottle of the animals' favorite liquid treat, Zoo Cola. The monkey drank  $\frac{1}{5}$  of the bottle. The zebra drank  $\frac{2}{3}$  of what was left. How much of the bottle of Zoo Cola did the zebra drink?

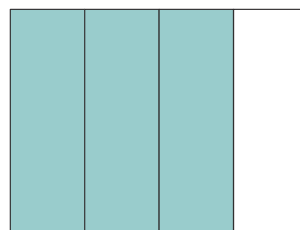


## Multiplication of Fractions

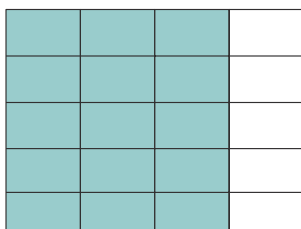
Area Model:  $\frac{3}{5} \times \frac{3}{4}$



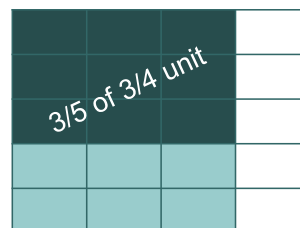
Assign unit for fraction.



Take  $\frac{3}{4}$  of UNIT



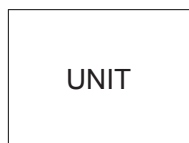
Subdivide unit into other fraction.



Answer is  $\frac{9}{20}$ .



## Multiplication of Fractions Area Model for $3\frac{2}{3} \times 2\frac{1}{4}$



This will be the unit for this problem.



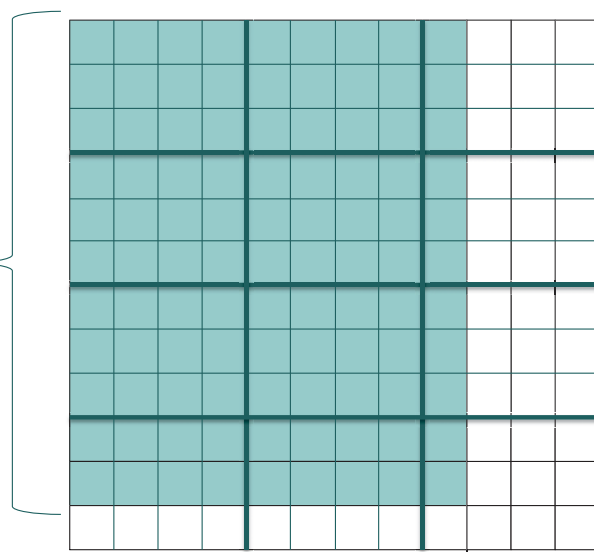
This is an area model for  $2\frac{1}{4}$ .



## Multiplication of Fractions

### Area Model for $3 \frac{2}{3} \times 2 \frac{1}{4}$

The shading represents  $2 \frac{1}{4}$  replicated  $3 \frac{2}{3}$  times.



The product is  **$\frac{99}{12}$**  which is equivalent to  **$8 \frac{1}{4}$** .



## Division of Fractions



Cassie has  $5 \frac{1}{4}$  yards of ribbon to make 3 bows for packages.

How much ribbon should she use for each bow if she wants to use the same length of ribbon for each package?





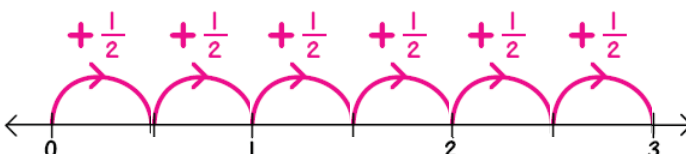


## Division Models

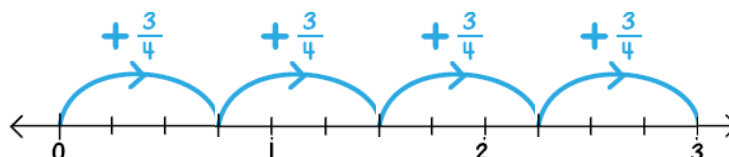


### Number Line Model

$$3 \div \frac{1}{2} =$$



$$3 \div \frac{3}{4} =$$



## Division Models



### Area Model for Division:

$\frac{3}{4}$   $\frac{2}{3} \rightarrow$  How many  $\frac{2}{3}$ s are there in  $\frac{3}{4}$ ?

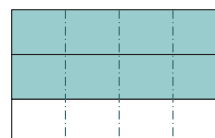
Unit to  
Begin

$\frac{3}{4}$  of unit



9 pieces  
in  $\frac{3}{4}$

$\frac{2}{3}$  of unit



8 pieces  
in  $\frac{2}{3}$

The new unit is the  
“ $\frac{2}{3}$  unit”,  
8 of the 12 pieces from the original unit.



## Division Models



**Area Model for Division:**

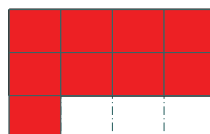
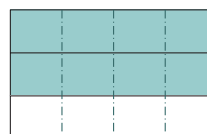
**$\frac{3}{4} \div \frac{2}{3} \rightarrow$  How many  $\frac{2}{3}$ s are there in  $\frac{3}{4}$ ?**

$\frac{3}{4}$  of unit



Rearrange the 9 pieces from  $\frac{3}{4}$  to cover the 8 pieces from  $\frac{2}{3}$ .

$\frac{2}{3}$  of unit



This makes one unit of  $\frac{2}{3}$  with one piece left over.

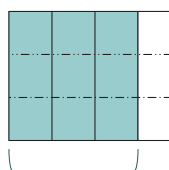
**ANSWER =  $1 \frac{1}{8}$**



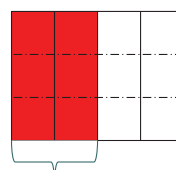
## Division of Fractions Area Model for $\frac{1}{2} \div \frac{3}{4}$



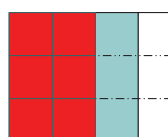
**How many  $\frac{3}{4}$ s in  $\frac{1}{2}$ ?**



$\frac{3}{4}$



$\frac{1}{2}$



The 6 pieces from “ $\frac{1}{2}$  part” fills  $\frac{2}{3}$  of the 9 pieces of the “ $\frac{3}{4}$  part.”

**ANSWER is  $\frac{2}{3}$ .**

## **Multiplication or Division**

Which of the following problems are solved by:

$1\frac{3}{4}$   $\frac{1}{2}$  OR  $1\frac{3}{4} \div \frac{1}{2}$  ?

1. How many cups of sugar do you need to make a half batch of cookies if a full batch takes  $1\frac{3}{4}$  cups of sugar?
2. How many posters can you paint with  $1\frac{3}{4}$  cans of paint if one poster takes  $\frac{1}{2}$  can of paint?
3. How many pounds of birdseed do you need to fill a bird feeder if  $1\frac{3}{4}$  pounds of birdseed fills the bird feeder  $\frac{1}{2}$  full?
4. What is the area, in square yards, of a rectangular garden that is  $1\frac{3}{4}$  yards long by  $\frac{1}{2}$  yard wide?
5. How many servings of lemonade can you make if you have  $1\frac{3}{4}$  cups of lemonade and a serving is  $\frac{1}{2}$  cup?

## **Division with Fractions**

Suppose a student suggests that when you divide fractions, you need only divide numerators and divide denominators.

Can we use this method for division with fractions? Will it always work?

$$a/b \div c/d = (a \div c) / (b \div d)$$

For example,  $8/15 \div 2/3 = (8 \div 2) / (15 \div 3) = 4/5$

$$1\frac{3}{4} \div \frac{1}{2} = 7/4 \div \frac{1}{2} = (7 \div 1) / (4 \div 2) = 7/2 = 3\frac{1}{2}$$



## Division with Fractions



**Do these methods  
always, sometimes, or never work?**

- Cross multiply and divide?
- Find a common denominator and then divide numerators?
- Add the dividend to itself the number of times as the denominator of the divisor and divide this sum by the numerator of the divisor?



## Division with Fractions



**Why does  
invert and multiply  
“work” to divide fractions?**

