DOINGWHATWORKS

SAMPLE MATERIAL

Teaching Students Math Problem-Solving Through Graphic Representations

Dr. Asha Jitendra, *Teaching Exceptional Children*, Vol. 34, No. 4

Topic: Response to Intervention in Elementary-Middle Math

Practice: Intentional Teaching

In this article, Dr. Jitendra describes the graphic representational technique, shows how to use the strategy for solving word problems, and discusses how to assess students' problem-solving performance.

This project has been funded at least in part with Federal funds from the U.S. Department of Education under contract number ED-PEP-11-C-0068. The content of this publication does not necessarily reflect the views or policies of the U.S. Department of Education nor does mention of trade names, commercial products, or organizations imply endorsement by the U.S. Government.

Teaching Students Math Problem-Solving Through Graphic Representations

Asha Jitendra

Lisa is an 8-year-old third grader who receives special education services in a learning support classroom for mathematics and reading. She has little confidence in her problem-solving ability and has a phobia for mathematics. When asked to solve word problems, Lisa typically grabs all the numbers in the problem and starts to add rather than pause to understand the nature of the problem she has to solve. Many students who take Lisa's approach to problem-solving—often students with learning disabilities and those at risk for math failure—become unsuccessful problem-solvers.

But here's the good news: We can use graphic representations to teach students with learning disabilities to be effective problem-solvers (see box, "What Is a Graphic Representational Technique?"). Recent studies have indicated that problem-solving instruction (e.g., graphic representational strategy) that emphasizes conceptual understanding can significantly help children with learning disabilities meet the challenges of the general education classrooms (Xin & Jitendra, 1999).

This article describes the graphic representational technique, shows how to use the strategy for solving word problems, and discusses how to assess students' problem-solving performance.

Teaching Problem-Solving Using the Graphic Representational Strategy

In addressing "big ideas" in mathematical problem-solving, we need to focus on carefully chosen problems. Here, we look at three different problem types, "change," "group," and "compare," that characterize most addition and subtraction problems presented in commercial basal mathematics programs (Marshall, 1990). Using graphic representations, students can develop their problem-solving skills, but also integrate addition and subtraction concepts. The essential elements of the graphic representation strategy are problem schemata identification and representation and problem solution.

Phase 1. Problem Schemata Identification and Representation

The goal of schema identification and representation instruction, a prerequisite to understanding and organizing information for later problem solution, is to facilitate conceptual understanding. *Problem schemata identification* involves recognizing the problem pattern (e.g., "change" or "group"), or semantic structure of the problem, whereas *problem representation* refers to translating a problem from words into a meaningful graphic representation.

The key aspects of this instruction are for students to

- Identify the separate features of each problem type (problems involving a change, a grouping, or a comparison—*change, group,* and *compare*).
- Organize and represent the relevant information in the story situation using schematic diagrams (see Figure 1).

Begin Phase 1 by providing students with story situations (see Figure 1) of each problem type. These situations present all quantities as *known* before students actually learn to solve word problems. Introduce problem-schema analysis using explicit modeling with several examples of story situations. During guided practice, use frequent student exchanges to facilitate the identification of critical elements of the

To guide students in memorizing the rules, provide them with a self-instructional sheet.

What Is a Graphic Representational Technique?

Effective word problem-solving requires the learner "to create a representation of the problem that mediates solution" (Goldman, 1989, p. 45). A primary characteristic of a graphic representational strategy that distinguishes it from other approaches is the use of schematic diagrams that allow students to organize information in the problem to facilitate problem translation and solution. The external representation (e.g., diagrams) may serve to reduce a learner's cognitive processing load and make available mental resources for engaging in problem analysis and solution.

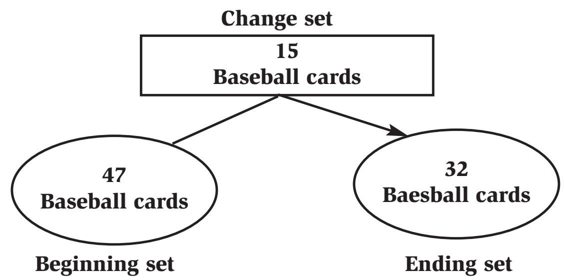
For example, the problem, "John has 3 marbles; Tim gives him 6 more marbles; how many marbles does John have now?" illustrates a beginning set, a "change" set, and an ending set requiring the selection of the *change* diagram. Based on the representation of key sets (i.e., beginning, change, and ending sets) in the problem, the problem-solver can select the appropriate arithmetic operation to find the quantity of the unknown set (i.e., ending) in the problem representation.

In contrast, though other instructional strategies also may include diagrams, the emphasis is less on identifying the semantic relations in the problem necessary for effective problem solution. In sum, the graphic representational strategy is a viable approach for enhancing students' conceptual knowledge about problem-solving, because it emphasizes instruction that goes beyond the mastery of algorithms used to perform operations to focus more on the semantic structure of problems.

Figure 1. Sample Story Situations and Schemata Diagrams for (a) Change, (b) Group, and (c) Compare Problem Type

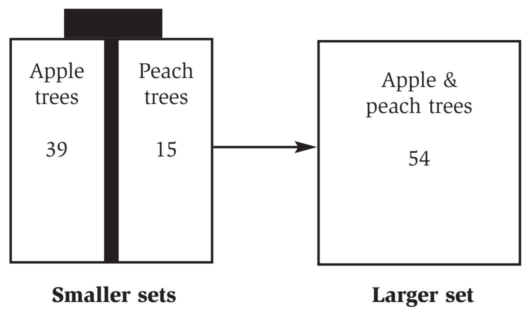
Change Story Situation

John had 47 baseball cards in his collection. He lost 15 of them when his family moved from Florida to New York. Now John has 32 baseball cards.



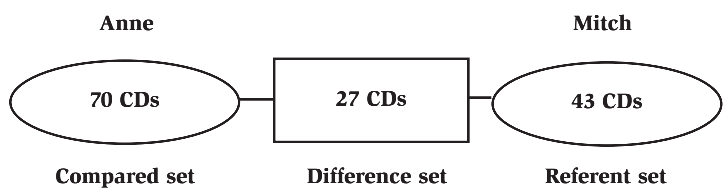
Group Story Situation

Tim has 54 fruit trees in his orchard. 39 are apple trees, and the remaining 15 are peach trees.



Compare Story Situation

Mitch has 43 CDs and Anne has 70. Anne has 27 more CDs than Mitch.



Source: From *Schemas in Problem Solving* (p. 135) by S. P. Marshall, 1995, New York: Cambridge University Press. Copyright 1995 by Cambridge University Press. Representation adapted by permission.

story. The following examples describe how to teach students to discern the features that govern each of the three different story situations:

- *"Change" story situation*. Consider the following story situation involving a change:

  John had 47 baseball cards in his collection. He lost 15 of them when his family moved from Florida to New York. Now John has 32 baseball cards.

Teach students that the "change" schema relation starts with a "beginning set" (e.g., 47 baseball cards) in which the object identity (e.g., baseball cards) and the amount of the object are defined (e.g., 47). Then a change (loses 15 baseball cards) occurs to the beginning set that results in an "ending set" (32 baseball cards) in which the new amount (i.e., 32) is defined. The ending set is less than the beginning set, because the word "lost" in the story situation implies that the change resulted in fewer baseball cards than the amount John initially had. Students learn that in this situation, both the beginning and ending sets could not occur at the same time; either there were 47 baseball cards or 32 baseball cards. Based on the position of the beginning, change, and ending sets in the story situation, students learn to infer the passage of time from past to present.

- *"Group" story situation*. Here is a sample story situation involving a group:

  Tim has 54 fruit trees in his orchard; 39 are apple trees, and the remaining 15 are peach trees.

Here, teach students that unlike the "change" relation, the passage of time is irrelevant to this "group" situation. The story did not say that Tim first had 39 apple trees and then planted 15 peach trees.

In addition, the "group" situation does not involve any change of object amounts. Instead, the group situation involves understanding part-whole relationships and knowing that the whole (i.e., superordinate) is equal to the sum of its parts (subordinates). Students learn that the subordinate categories (apple trees and peach trees) should have semantic ties to the superordinate category (trees).

For example, students learn the common attributes of the three elements, "apple trees," "peach trees," and "trees," and the relationships among them in the story situation. Although "apple trees" and "peach trees" are instances of trees with some similar attributes, they are distinct in that an apple tree is not the same as a peach tree.

- *"Compare" story situation*. Here is a story situation involving a comparison, or a "compare" relation:

> Mitch has 43 music CDs and Anne has 70. Anne has 27 more music CDs than Mitch.

Teach students to focus on the presence of two sets (Mitch and Anne and their music records), each associated with a smaller or larger value (i.e., 43 and 70) and both having the same unit of measure (e.g., music CDs). One set serves as the comparison set (Anne's music CDs) and the other as the referent set (Mitch's music CDs). Typically, students learn to identify the compared and referent sets and determine the larger set by examining the comparison statement (i.e., Anne has 27 more music CDs than Mitch). Students focus on the *more than* or *less than* concepts to *compare* the two sets and identify the difference in value between the two sets.

For each problem type, the teacher or students read the story situation and map features of the situation onto the appropriate schematic diagrams. Initially, worksheets should include story situations of a single problem type. When students learn to identify and map subsequent problem types, worksheets should gradually include all three problem types. This phase should continue until students are able to accurately discern the three different problem types.

Phase 2. Problem Solution

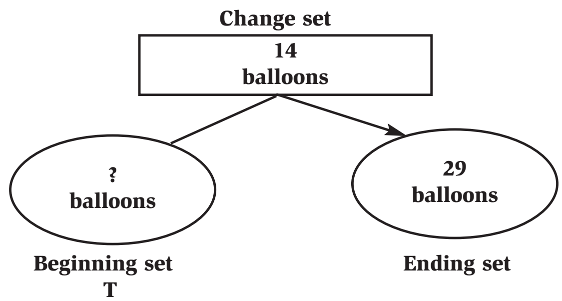
Problem solution includes the selection and application of appropriate mathematical operations based on the problem representation. The key aspects of problem solution instruction are for students (a) to plan to solve the problem by identifying the action procedure (e.g., counting, adding, subtraction) and sequence of steps; and (b) to carry

Using graphic representations to emphasize conceptual understanding can help children with learning disabilities in inclusive classrooms.

Figure 2. Sample Procedure for Solving (a) Change, (b) Group, and (c) Compare Problems

Change Problem

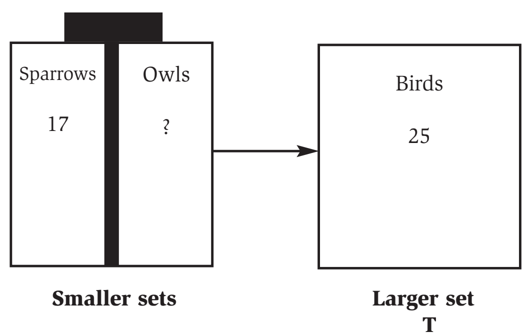
A balloon man had some balloons. Then 14 balloons blew away and the man now has 29 balloons. How many balloons did the man begin with?



Total is not known, so add.
29 + 14 = 43
The man began with 43 balloons.

Group Problem

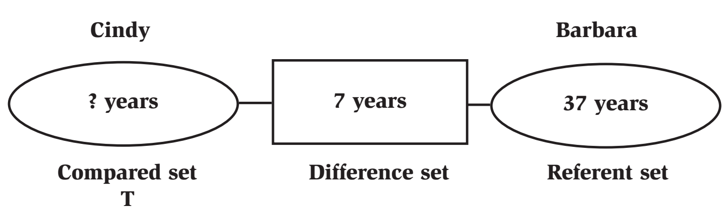
Jenny saw 25 birds on a camping trip. She saw 17 sparrows and some owls. How many owls did Jenny see on the camping trip?



Total is known, so subtract.
25 - 17 = 8
Jenny saw 8 owls on the camping trip.

Compare Problem

Barbara is 37 years old. Cindy is 7 years older than Barbara. How old is Cindy?



Total is not known, so add.
37 + 7 = 44
Cindy is 44 years old.

out the operation identified in the planning step to solve the problem.

Once students display adequate knowledge of problem schemata, teach them to solve word problems. Begin this phase with a review of the problem schemata within the context of *real* word problems, rather than the story situations shown in Phase 1. Through a question-and-answer session, prompt students to identify the problem type and the key features of the problem. Then ask them to map the information onto the relevant diagram and flag the missing information in the problem, using a question mark. During this phase, address any misconceptions with explicit feedback and additional modeling.

After students identify and map the information onto diagrams, they learn to plan to solve the problem. They begin by finding the "whole" or total amount in the word problem by focusing on the specific information provided in the text, as follows:

- In a "change" problem, students learn whether the problem ends up with *more* or *less* than the beginning amount. Teach students that when the problem ends up with *more*, the ending amount represents the total. If the problem ends up with *less*, however, the beginning amount is the total.
- For the "group" problem, the larger group object always represents the total, because the smaller groups combine to form the larger group.
- To find the total amount in the "compare" problem, students determine whether the *referent* or the *compared* set represents the higher value. Teach students to examine the comparison, or "difference" statement, to find the set that represents the total amount in the "compare" problem. In each instance, students learn to write *T* for total under the specific set that represents the total (see Figure 2).

Next, students select an arithmetic operation based on which *part* of the problem situation is unknown and which of the critical elements in the problem structure represents the total or *whole*. Finally, they solve the problem

Figure 3. Rules to Identify the Total and the Operation for (a) Change, (b) Group, and (c) Compare Problems

Finding the Total

a. "Change" Problem
If the problem ends up with more than it started with, then the ending set is the total. If the problem ends up with less than it started with, then the beginning set is the total.

b. "Group" Problem
The larger group set is always the total.

c. "Compare" Problem
The larger set (compared or referent) in the comparison or difference statement is the total.

Figuring Out the Operation (Addition or Subtraction)

When the total is unknown, *add* to find the total. When the total is known, *subtract* to find the other amount.

Figure 4. Representational Strategy

Problem Schemata Identification and Representation

1. Find the problem pattern
   (a) Read the problem carefully
   (b) Ask whether the problem is a change, group, or compare problem type
2. Organize and represent the information in the problem using schemata diagrams
   (a) Map the known information (object identity and object amount) onto the schema diagram
   (b) Flag the unknown information using a question mark

Problem Solution

3. Plan to solve the problem
   (a) Find the object identity that represents the "whole" or total amount and write a "T" for total under the set
   (b) Select an arithmetic operation based on the known and unknown information (i.e., "When the total is not known, add to find the total; when the total is known, subtract to find the part.")
4. Solve the problem
   (a) Add or subtract
   (b) Check if the answer makes sense
   (c) Write the whole answer

Schematic diagrams allow students to organize information in the problem to facilitate problem translation and solution.

using a generalizable rule, "When the total is not known, add to find the total; when the total is known, subtract to find the part or other amount," which is based on the part-whole concept (see Figure 2). To guide students in memorizing the rules, provide them with a self-instructional sheet with written rules for identifying the total and determining the operation needed to solve the problem (see Figure 3). The note sheet can serve as a scaffold until students can independently verbalize the rules. The self-instructional note sheet and diagrams serve to improve students' understanding and memory of information in word problems. However, once students are proficient in using the schemata diagrams, fade or remove them and encourage them to represent the problem using their own diagrams. Figure 4 presents the instructional sequence of the representational strategy.

Evaluating Problem-Solving Performance

When teaching the graphic representational strategy, ask students to complete word problem tests at the end of each instructional session to check their understanding and to evaluate whether they have mastered the strategy steps. Examine students' completed tests not only for correct solutions but also for strategy use (e.g., draw a diagram, map information onto the diagram, plan to solve the problem by figuring out the operation to use and the sequence of steps) and provide them with additional instruction, as needed, before they move to the next problem type.

Initially, assess students' performance in solving one problem type. Later, when they complete instruction in the use of the strategy steps for all problem types, present tests that include all three problem types. In addition, evaluate how well students maintain the skill and knowledge after the strategy lessons are over; and discover how well they have transferred the skills to novel and complex problems.

Student Success

My colleagues and I have successfully taught the graphic representational strategy to students with disabilities in elementary and middle schools in our research investigations (Jitendra et al., 1998; Jitendra & Hoff, 1996; Jitendra, Hoff, & Beck, 1999). We found dramatic improvements in students' problem-solving scores. In addition, they maintained the skills for as long as 4 weeks, and students were able to use the strategy in novel and complex problems. In these studies, we also found that the performance of students with learning disabilities either approached or surpassed that of a sample of students without disabilities.

Students also reported positive attitudes regarding the strategy instruction. For example, students' responses about what they liked the most about the strategy were as follows: "Learning to solve problems," "Getting the correct answers," "It's easier to understand, it gave me clues," "I liked the problems," "It was fun," "They were cool." When asked if they would recommend this strategy to a friend, they all answered in the affirmative. One student even reported that she felt she could go to college and become a teacher, which she was not sure she would be able to do before participating in our study. Further, this student was so pleased with her success that she became involved in tutoring a younger student in a general education classroom using the strategy.

Teachers of students in our studies also have indicated transfer of effective problem-solving skills into their classrooms. One teacher noted that the student who evidenced math phobia before the intervention showed increased confidence in her math performance and was more consistent in labeling her work on problem-solving tasks.

Final Thoughts

This article shows that we *can* teach students with learning disabilities to use effective problem-solving skills. We should not have to wait until students with learning disabilities master easier material (e.g., computation skills) before presenting them with problem-solving opportunities.

References

Goldman, S. (1989). Strategy instruction in mathematics. *Learning Disability Quarterly, 12*, 43-55.

Jitendra, A. K., Griffin, C., McGoey, K., Gardill, C., Bhat, P., & Riley, T. (1998). Effects of mathematical word problem solving by students at risk or with mild disabilities. *Journal of Educational Research, 91*, 345-356.

Jitendra, A. K., & Hoff, K. (1996). The effects of schema-based instruction on mathematical word problem solving performance of students with learning disabilities. *Journal of Learning Disabilities, 29*, 422-431.

Jitendra, A. K., Hoff, K., & Beck, M. (1999). Teaching middle school students with learning disabilities to solve multistep word problems using a schema-based approach. *Remedial and Special Education, 20*(1), 50-64.

Marshall, S. (1990). The assessment of schema knowledge for arithmetic story problems: A cognitive science perspective. In G. Kulm (Ed.), *Assessing higher order thinking in mathematics*. Washington, DC: American Association for the Advancement of Science.* (ERIC Document Reproduction Service No. ED 324 201)

Marshall, S. P. (1995). *Schemas in problem solving*. New York: Cambridge University Press.

Xin, Y. P., & Jitendra, A. K. (1999). The effects of instruction in solving mathematical problems for students with learning problems: A meta-analysis. *The Journal of Special Education, 32*(4), 207-225.

Asha Jitendra *(CEC Chapter #905), Associate Professor, Education and Human Services, Lehigh University, Bethlehem, Pennsylvania.*

Address correspondence to the author at Education and Human Services, Lehigh University, 111 Research Drive, Bethlehem, PA 18015-4794 (e-mail: akj@lehigh.edu).

TEACHING Exceptional Children, *Vol. 34, No. 4, pp. 34-38.*